BANGABASI EVENING COLLEGE

19, Rajkumar Chakraborty Sarani, Kolkata- 700 009, West Bengal

SYLLABUS
for
M.Sc. in MATHEMATICS
Four Semesters

Effective from the academic session 2015 -2016 and onwards
# Syllabus: M.Sc. (Mathematics)

## First Year First Semester

*Total marks: 250  
Total No. of Lectures: 50 Hours per 50 Marks*

<table>
<thead>
<tr>
<th>Paper Code</th>
<th>Paper Name</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>MA101</td>
<td>Linear &amp; Multilinear Algebra</td>
<td>50</td>
</tr>
<tr>
<td>MA102</td>
<td>Real Analysis &amp; Multivariable Calculus</td>
<td>50</td>
</tr>
<tr>
<td>MA103</td>
<td>Complex Analysis</td>
<td>50</td>
</tr>
<tr>
<td>MA104</td>
<td>Ordinary Differential Equations</td>
<td>50</td>
</tr>
<tr>
<td>MA105</td>
<td>Classical Mechanics</td>
<td>50</td>
</tr>
</tbody>
</table>
## First Year Second Semester

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<tr>
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<tbody>
<tr>
<td>MA201</td>
<td>Algebra</td>
<td>50</td>
</tr>
<tr>
<td>MA202</td>
<td>General Topology</td>
<td>50</td>
</tr>
<tr>
<td>MA203</td>
<td>Partial Differential Equations &amp; Integral Transform</td>
<td>50 (26+24)</td>
</tr>
<tr>
<td>MA204</td>
<td>Optimization &amp; Graph Theory</td>
<td>50     (26+24)</td>
</tr>
<tr>
<td>MA205</td>
<td>Measure Theory &amp; Probability</td>
<td>50     (26+24)</td>
</tr>
</tbody>
</table>
# Second Year First Semester

*Total marks: 250*

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<tr>
<th>Paper Code</th>
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<tbody>
<tr>
<td>MA301</td>
<td>Functional Analysis &amp; Lie Groups and Lie Algebras</td>
<td>50 (38+12)</td>
</tr>
<tr>
<td>MA302</td>
<td>Numerical Analysis (Theory) &amp; Computational Mathematics (Practical)</td>
<td>50 (24+26)</td>
</tr>
<tr>
<td>MA303</td>
<td>Special Paper (one from first four and one from last four)</td>
<td>100 (50+50)</td>
</tr>
<tr>
<td>MA304</td>
<td>Differential Geometry &amp; Theory of Manifolds</td>
<td>50</td>
</tr>
</tbody>
</table>
List of special papers (each has a sequence of two papers):

1. Advanced Real Analysis (MA303A & MA403A)
2. Advanced Complex Analysis (MA303B & MA403B)
3. Applied Functional Analysis (MA303C & MA403C)
4. Advanced Graph Theory (MA303D & MA403D)
5. Advanced Operation Research (MA303E & MA403E)
6. Fluid Mechanics (MA303F & MA403F)
7. Plasma Dynamics (MA303G & MA403G)
8. Advanced Probability & Stochastic Processes (MA303H & MA403H)
## Second Year Second Semester

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<tr>
<td>MA401</td>
<td>Dynamical System &amp; Elements of Fuzzy Set Theory</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(38+12)</td>
</tr>
<tr>
<td>MA402</td>
<td>Continuum Mechanics &amp; Integral Equations</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(26+24)</td>
</tr>
<tr>
<td>MA403</td>
<td>Special Paper (sequence continued from the 3rd semester)</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(50+50)</td>
</tr>
<tr>
<td>MA404</td>
<td>Dissertation, Internal Assessment, Seminar &amp; Grand Viva</td>
<td>50</td>
</tr>
</tbody>
</table>
Syllabus: PG Mathematics, BEC

Detailed Syllabi

MA101: Linear & Multilinear Algebra

Vector spaces over fields, subspaces, bases and dimension, existence of basis. Examples of infinite dimensional vector spaces, quotient space.

Systems of linear equations, matrices, rank, Gaussian elimination.

Linear transformations, representation of linear transformations by matrices, rank-nullity theorem, duality, bi-dual and transpose.

Eigenvalues and eigenvectors, characteristic polynomials, minimal polynomials, Cayley-Hamilton Theorem, primary decomposition theorem, triangular decomposition, diagonalization, singular value decomposition, rational canonical form, Jordan canonical form.

Inner product spaces, Gram-Schmidt orthonormalization, orthogonal projections, linear functionals and adjoints, Hermitian, selfadjoint, unitary and normal operators, Spectral Theorem for normal operators.

Bilinear forms, symmetric and skewsymmetric bilinear forms, real quadratic forms, Sylvester's law of inertia, positive definiteness. Applications to Geometry & Mechanics.

Introductory concepts of multilinear algebra and its applications.

Texts / References:


MA102: Real Analysis

&

Multivariable Calculus

Real Analysis

Pre-requisites:
Structure of Real line, Concept of denumerable and non-denumerable sets, continuity, differentiability, sequence and series of functions, uniform convergence, Riemann integral, functions of bounded variation

Cardinal number:
Concept of Cardinal numbers of an infinite Set, Ordering of the set of Cardinal numbers, Schr¨oder-Bernstein theorem, arithmetic of Cardinal numbers, Axiom of choice, Continuum hypothesis.

Lebesgue measure:
Algebra and σ- algebra of sets, σ- algebra generated by a class of subset, Borel sets, Lebesgue outer measure on R, Caratheodory Extension Procedure, Measureability and measure, Approximation of Lebesgue measure by open and closed sets, Existence of non-measurable sets, Cantor Sets.

Functions in R:
Convex functions and its simple properties, Absolutely Continuous functions on [a,b] and its elementary properties, Invariance of measurability under absolutely continuous functions, Weierstrass Approximation theorem,

Derivatives :
Concept of Vitali-covering and statement of Vitali-covering theorem. Applications of Vitali-covering theorem on the derivative of absolutely continuous functions, Weierstrass non-differentiable functions , Notion of Dini’s derivates and its simple properties

Integrations:
Lebesgue’s criterion of Riemann integrability over a bounded function on \([a, b]\) and its consequences, Concept of Riemann-Stieltjes integral on \([a, b]\) and its simple properties, Integration by parts, Stieltjes integral as a Riemann integral, Step function as integrator.

**Multivariate Calculus:**

Concept of limits and continuity of functions from \(\mathbb{R}^n\) to \(\mathbb{R}^m\) : directional derivatives, differential or Frechet derivative, properties of differential, Mean-value theorems, Jacobian, Inverse Function Theorem, Implicit Function Theorem.

**Texts / References:**


W. Sierpinski, *Cardinal and ordinal number*. 
MA103: Complex Analysis

Complex Integration, line integral and its fundamental properties, Cauchy’s fundamental theorem, Cauchy’s integral formula and higher derivatives, power series expansion of analytic functions.


Argument principle, Rouche’s theorem and its application.

Maximum modulus theorem.

Conformal mappings, Schwarz’s Lemma, Schwarz, s -Ahlfors-Pick Theorem and its consequence.

Introduction to Analytic continuation.

Texts / References:
A.R. Shastri, An Introduction to Complex Analysis, Macmilan India, New Delhi, 1999.
MA104: Ordinary Differential Equations

Existence of solution near an ordinary point and a regular singular point. Power series solutions about an ordinary point. Solution about singular points, The Method of Frobenius. Solution of Bessel and Legendre equation.

Bessel’s functions and its generating function, recurrence relation.

Legendre polynomials, generating function, recurrence relation, Rodrigue,s formula, Laplace,s integral formula, orthogonal property.


Adjoint and Self-adjoint poerators. Eigenfunction expansions. Strum-Liouville theory.

Inhomogeneous two point boundary value problem ( Ly=f); Wronskian and variation of parameters. Concept and use of Green’s functions.

Texts / References:


G F Simmons, TATA McGraw – Hill Differential Equation with applications and historical notes.

Shepley L Ross, Wilet India (P) Ltd Differential Equation

Stakgold (John Wiley & Sons.), Greens Functions and Boundary Value Problems

Murray, Daniel A, Differential Equation

J. D. Logan, Applied Mathematics, (3rd Ed. Wiley Interscience)
MA105: Classical Mechanics


Generalised co-ordinates: Degrees of freedom, Constraint, Principle of Virtual Work.

Lagrangian formulation of Dynamics: Lagrange’s equations of motion for holonomic and non-holonomic systems. Ignorable coordinates, Constants of motion, Brachistochrone problem.

Principle of Stationary Action or Least Action, Hamilton principle, Calculus of variations.

Invariance transformations, Noether’s theorem, Space time transformations.

Hamilton’s Equations, plane pendulum problem, Poisson brackets. Canonical transformations.

Hamilton-Jacobi theory, Free particle problem, Time-independent Hamilton-Jacobi equation, harmonic oscillator case.


Motion of a particle relative to rotating earth. Coriolis force, Foucault’s pendulum.

Texts / References:

F. Chorlton – A Text Book of Dynamic

Synge and Griffith – Principles of Mechanics

D. T. Green Wood – Classical Dynamics

E. T. Whittaker – A Treatise on the Analytical Dynamics of Particles and Rigid Bodies

K. C. Gupta – Classical Mechanics of Particles and Rigid Bodies

F. Gantmacher – Lectures in Analytical Mechanics

H. Goldstein – Classical Mechanics
MA201: Algebra

Cyclic groups, generators and relations, Cayley's Theorem, group actions, Sylow Theorems. Direct products, Structure Theorem for finite abelian groups.

Simple groups and solvable groups, nilpotent groups, simplicity of alternating groups, composition series, Jordan-Holder Theorem. Semidirect products. Free groups, free abelian groups.

Rings, Examples (including polynomial rings, formal power series rings, matrix rings and group rings), ideals, prime and maximal ideals, rings of fractions, Chinese Remainder Theorem for pairwise comaximal ideals. Euclidean Domains, Principal Ideal Domains and Unique Factorizations Domains. Polynomial rings over UFD's.

Fields, Characteristic and prime subfields, Field extensions, Finite, algebraic and finitely generated field extensions, Classical ruler and compass constructions, Splitting fields and normal extensions, algebraic closures. Finite fields, Cyclotomic fields, Separable and inseparable extensions.

Galois groups, Fundamental Theorem of Galois Theory, Composite extensions, Examples (including cyclotomic extensions and extensions of finite fields).

Norm, trace and discriminant. Solvability by radicals, Galois' Theorem on solvability.

Cyclic extensions, Abelian extensions, Polynomials with Galois groups Sn. Transcendental extensions.

Texts / References:


Hungerford, Abstract Algebra

Malik, Mordeson, Sen- Abstract Algebra
MA202: General Topology

General Topology: Topological Spaces: open sets, closed sets, neighbourhoods, bases, subbases, limit points, closures, interiors, continuous functions, homeomorphisms.

Examples of topological spaces: subspace topology, product topology, metric topology, order topology.

Quotient Topology: Construction of cylinder, cone, Moebius band, torus, etc.

Connectedness and Compactness: Connected spaces, Connected subspaces of the real line, Components and local connectedness, Compact spaces, Heine-Borel Theorem, Local -compactness.


Equicontinuity, Ascoli-Arzela Theorem, Baire Category Theorem.

Texts / References:


MA203: Partial Differential Equations

&

Integral Transform

Partial Differential Equations:

First order pdes: Cauchy problem for the linear first order equation and characteristics, linear and quasi-linear pdes.

Second order pdes: Classifications, fundamental solutions: Laplace, Wave and Diffusion equations.

Different methods of solving them: Methods of separation of variables for heat, Laplace and wave equations, Fourier transform method, Laplace transform method, method of characteristics, d’Alembert’s solution to the wave equation and propagation of discontinuities, introduction to Green's function method. Maximum-minimum principles.


Integral transform:

Fourier Transform and its properties, Inversion formula of F.T.; Convolution Theorem; Parseval’s relation. Applications.

Laplace’s Transform and its properties. Inversion by analytic method and by Bromwitch path. Lerch’s Theorem. Convolution Theorem; Applications.

Mellin and Hankel Transforms and their inverse. Application to Boundary value problems.


Texts / References:


I. N. Sneddon – Fourier Transforms (MacGraw-Hill)

R. V. Churchill – Operational Methods.

Andrews & Shivamoggi : Integral transforms for Engineers.

L. Debnath & D. Bhatta : Integral Transforms and Their Applications.

MA204: Optimization

&

Graph Theory

Unconstrained optimization using calculus (Taylor's theorem, convex functions, coercive functions).

Unconstrained optimization via iterative methods (Newton's method, Gradient/ conjugate gradient based methods, QuasiNewton methods).

Constrained optimization (Penalty methods, Lagrange multipliers, Kuhn-Tucker conditions. Linear programming (Simplex method, Dual simplex, Duality theory). Modeling for Optimization.

Graph Theory: Basic Concepts, various kinds of graphs, simple graphs, complete graph, walk, tour, path and cycle, Eulerian graph, bipartite graph (characterization), Havel-Hakimi theorem and Erdos-Gallai theorem (statement only), hypercube graph, Petersen graph, trees, forests and spanning subgraphs, distances, radius, diameter, center of a graph, the number of distinct spanning trees in a complete graph.

Trees: Kruskal and Prim algorithms with proofs of correctness, Dijkstra’s a algorithm, Breadth first and Depth first search trees, rooted and binary trees, Huffman’s algorithm

Matchings: augmenting path, Hall’s matching theorem, vertex and edge cover, independence number and their connections, Tutte’s theorem for the existence of a 1-factor in a graph

Planar graphs Embedding a graph on plane, Euler’s formula, non-planarity of K5 and K3,3, classification of regular polytopes, Kuratowski’s theorem (no proof).

Texts / References:


Syllabus: PG Mathematics, BEC


N. Deo (Prentice-Hall, 1974), Graph Theory.
MA205: Measure Theory & Probability

Measure Theory: Semi-algebra, Algebra, Monotone class, Sigma-algebra, Monotone class theorem. Measure spaces.

Outline of extension of measures from algebras to the generated sigma-algebras: Measurable sets; Lebesgue Measure and its properties.

Measurable functions and their properties; Integration and Convergence theorems.


Product measure spaces, Fubini’s theorem.

Fundamental Theorem of Calculus for Lebesgue Integrals (an outline).

Probability: Probability measure, probability space, construction of Lebesgue measure, extension theorems, limit of events, Borel-Cantelli lemma.

Random variables, Random vectors, distributions, multidimensional distributions, independence.

Expectation, change of variable theorem, convergence theorems (can be just mentioned as it is a repetition of above in measure theory part).

Sequence of random variables, modes of convergence and implications. Moment generating function and characteristics functions, inversion and uniqueness theorems, continuity theorems, Weak and strong laws of large number, central limit theorem, Kolmogorov three series theorem, 0-1 laws.

Radon Nikodym theorem, definition and properties of conditional expectation, conditional distributions and expectations.

Texts / References:


